IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Comment on the optical Dicke model

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1975 J. Phys. A: Math. Gen. 8 L115

(http://iopscience.iop.org/0305-4470/8/10/007)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.88
The article was downloaded on 02/06/2010 at 05:02

Please note that terms and conditions apply.

J. Phys. A: Math. Gen., Vol. 8, No. 10, 1975. Printed in Great Britain. © 1975

LETTER TO THE EDITOR

Comment on the optical Dicke model

B V Thompson

Department of Mathematics, UMIST, PO Box 88, Manchester M60 1QD, UK

Received 13 August 1975

Abstract. The classical part of the dipole–dipole interaction prevents the super-radiant phase transition in a dielectric crystal.

The Hamiltonian of the generalized Dicke model is usually written

$$\sum_k \omega_k a_k^\dagger a_k + \Omega_0 \sum_l \sigma_3(l) + 2 \sum_k \sum_l G_k(a_k \,\mathrm{e}^{\mathrm{i}\boldsymbol{k}.\boldsymbol{l}} + a_k^\dagger \,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}.\boldsymbol{l}})\sigma_1(l) \tag{1}$$

where $\sigma_{1,3}(l)$ are S-spin operators for atoms on sites labelled $\boldsymbol{l}$ and $a_k, a_k^\dagger$ are the boson field operators. The significance of the parameters Ω_0, G_k is the point of this article. In the optical case, the interaction term is meant to represent the coupling of the transverse electric field and atomic dipole moments. When the vectors $\boldsymbol{l}$ form a crystal lattice structure it is possible to obtain the dispersion relation for travelling waves in the linear optics limit. This involves replacing $2\sigma_1(l)$ by $(2S)^{1/2}(b_l + b_l^\dagger)$ using the Holstein–Primakoff formulae. The operators $b_l, b_l^\dagger$ are localized exciton operators. By a standard procedure, we find for the dispersion relation

$$\nu^4 - (\omega_k^2 + \Omega_0^2)\nu^2 + \omega_k^2\Omega_0^2 = 8G_k^2 SN\Omega_0\omega_k. \tag{2}$$

This is not the expected expression for a dielectric. It has the disturbing property of admitting the solution

$$\nu = 0, \qquad \text{if } 8G_k^2 SN = \omega_k\Omega_0.$$

The very least that we ought to expect from the optical Dicke model is that it should yield the classical dielectric Hamiltonian in the linear optics limit, ie the dispersion relation should be

$$\nu^4 - (\omega_k^2 + \omega_0^2\epsilon)\nu^2 + \omega_k^2\omega_0^2 = 0 \tag{3}$$

where $\hbar\omega_0$ is the atomic energy level spacing and ϵ is the static dielectric constant (Hopfield 1958).

Agreement between (2) and (3) is assured if we take

$$\Omega_0 \equiv \omega_0\epsilon^{1/2}; \qquad 8G_k^2 SN\Omega_0 \equiv \omega_0^2\omega_k(\epsilon - 1). \tag{4}$$

By this re-interpretation of Ω_0 and G_k, we have effectively included an important part of the transverse dipole–dipole interaction in the Hamiltonian (1) (see eg Kittel 1963, p 44–6).

Wang and Hioe (1973) give the condition for a super-radiant phase transition to exist in the long-wavelength limit as ($S = \frac{1}{2}$)

$$\sum_k \frac{4NG_k^2}{\omega_k \Omega_0} > 1.$$

With the amended interpretation of Ω_0 and G_k given by (4), this becomes

$$(1-\epsilon^{-1})\sum_k 1 > 1 \qquad (5)$$

which shows that a phase transition is impossible with just one field mode. When more modes are involved, (5) can be satisfied but is valid only for very long waves.

It has been proved that for a set of S-spins in any space configuration having a centre of inversion symmetry, the inequality

$$\sum_k \frac{8G_k^2 SN}{\omega_k \Omega_0} > 1$$

is merely a necessary condition for the existence of the phase transition (Thompson 1975, to be submitted for publication). Only in the very long-wave limit is it also sufficient.

If the wavelength is considerably less than the system dimensions and we have a lattice structure, the phase transition requirement with modes $\boldsymbol{k}$ and $-\boldsymbol{k}$ is $1-\epsilon^{-1} > 1$ rather than $2(1-\epsilon^{-1}) > 1$ given by (5), so that no transition can occur in this case either. In fact one can consider four or six modes of equal frequency and directions dependent on the lattice symmetry, but the transition is still lost (Thompson 1975, to be submitted for publication).

A recent suggestion that dipole–dipole interactions ought to be considered in non-linear optics problems (de Martini 1974) appears to be strongly supported by the above elementary analysis.

References

Hopfield J J 1958 *Phys. Rev.* **112** 1555–67

Kittel C 1963 *Quantum Theory of Solids* (New York: Wiley)

de Martini F 1974 *Lett. Nuovo Cim.* **10** 275–80

Wang Y K and Hioe F T 1973 *Phys. Rev.* A **7** 831–6